Single Transverse Spin Asymmetry : Theoretical Overview

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Content:

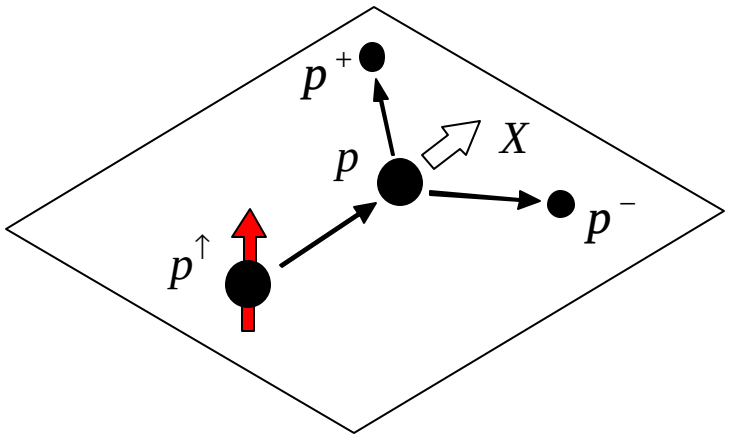
1. Introduction: Basics for single spin asymmetry (SSA) Data, Ingredients of SSA, “T-odd” observable, SSA and chiral symmetry breaking.
2. T-odd distribution and fragmentation functions.
3. Twist-3 effects
4. Summary

1. Introduction.

Single Spin Asymmetry (SSA)

- $p^{\uparrow}p \to \pi X$

$$A_N = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$



E704('91) and STAR('02) found big asymmetry in the forward direction ! (large x_F)

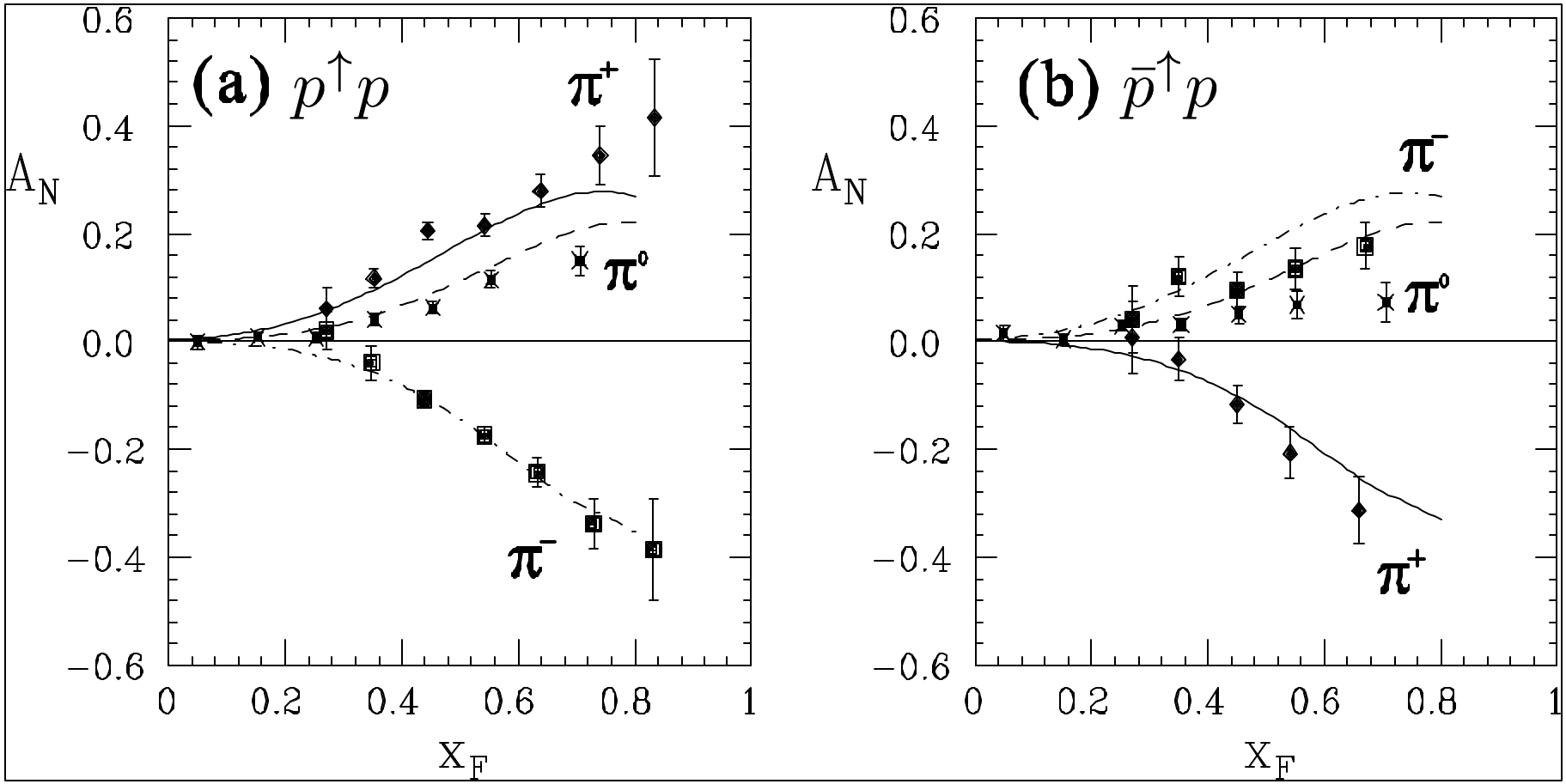
- $pp \to \Lambda^{\uparrow}X$; experimentally known since 70's.

  Bunce et al.('76), Heller et al.('78)....

- SSA in SIDIS;
  $ep^{\uparrow} \to e'\pi X$, HERMES('99,'03); $\nu p \to \mu\Lambda^{\uparrow}X$, NOMAD('00); $ep \to e\Lambda^{\uparrow}X$, (eRHIC)..

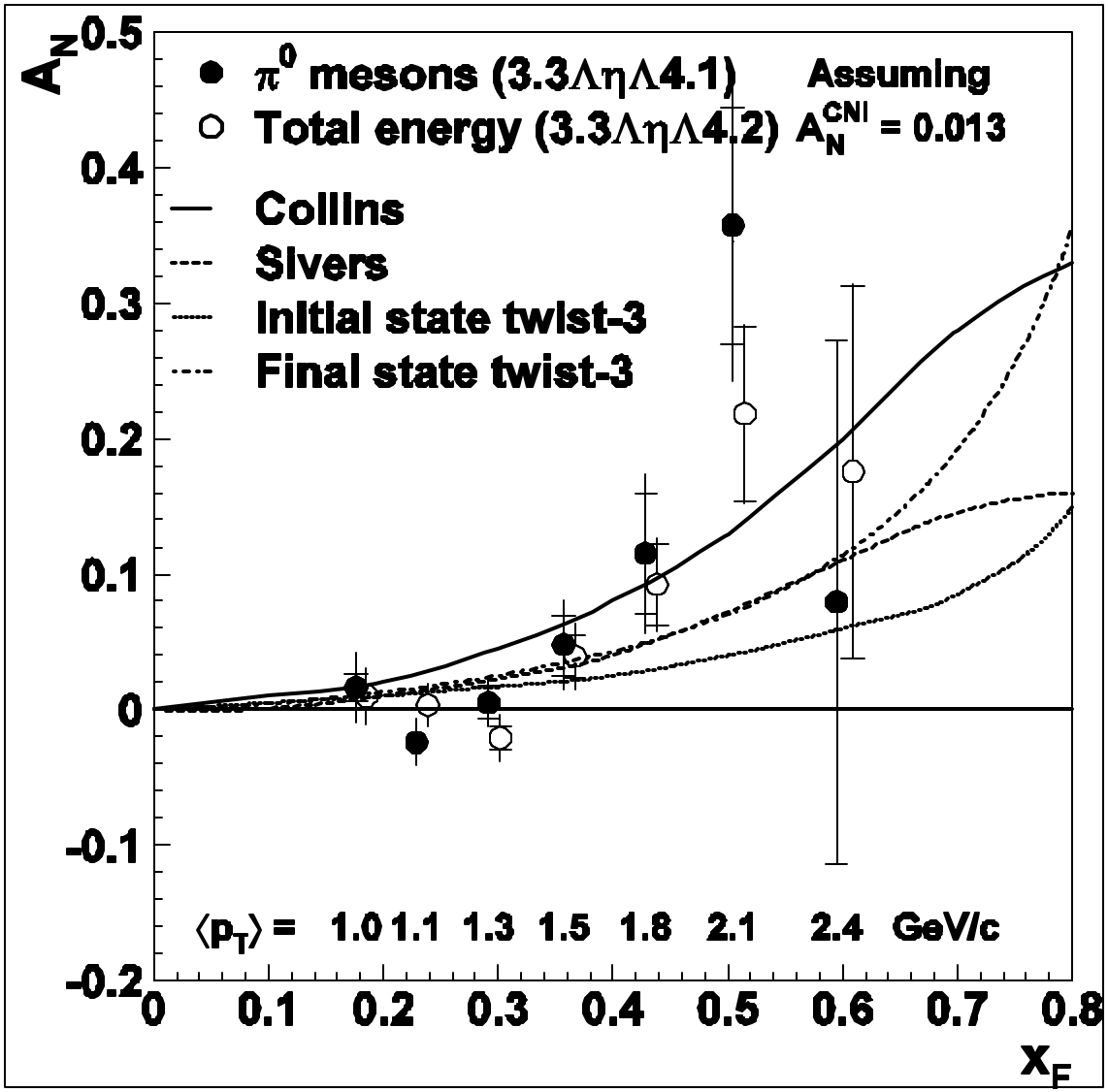
FNAL-E704: A_N for $p^\uparrow p \to \pi X$ and $\bar{p}^\uparrow p \to \pi X$

$\sqrt{s} = 20$ GeV, $p_T = 1.5$ GeV



$x_F = 2p_{\|}/\sqrt{s}$

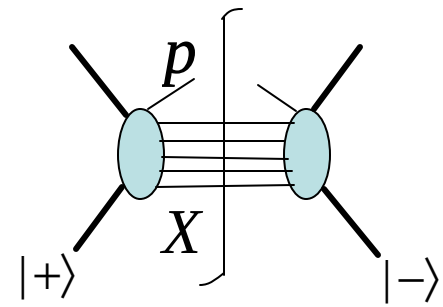
RHIC-STAR, hep-ex/0310068



Fundamentals of SSA: ex. $p^\uparrow p \to \pi X$

$|\uparrow(\downarrow)\rangle = \frac{1}{\sqrt{2}}(|+\rangle \pm i|-\rangle)$: $\perp$ pol. states with the helicity eigenstates $|\pm\rangle$.

$$\begin{aligned}\sigma^\uparrow - \sigma^\downarrow &= \sum |\langle\cdots|T|\uparrow\rangle|^2 - \sum |\langle\cdots|T|\downarrow\rangle|^2 \\ &= 2\,\mathrm{Im}\left\{\sum \langle -|T^\dagger|\cdots\rangle\langle\cdots|T|+\rangle\right\}.\end{aligned}$$



∗ Ingredients for SSA

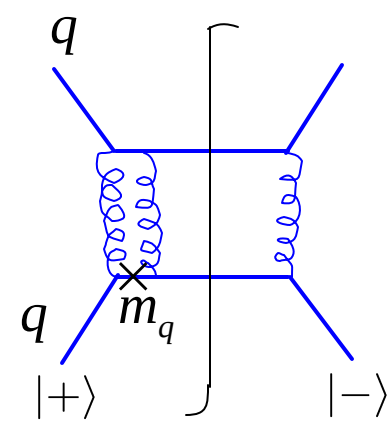
(i) Helicity flip in the forward (cut) amplitude denoted as { }.

(ii) $\langle\cdots|T|-\rangle$ and $\langle\cdots|T|+\rangle$ should have different phases.

∗ BUT, in conventional twist-2:

(i) is caused only by $m_q \neq 0$ in the hard cross section.

(ii) is caused only through loop corrections to the hard cross section.



—— $A_N \sim \frac{\alpha_s m_q}{p_T}$: small! (Kane *et al.* ('78))

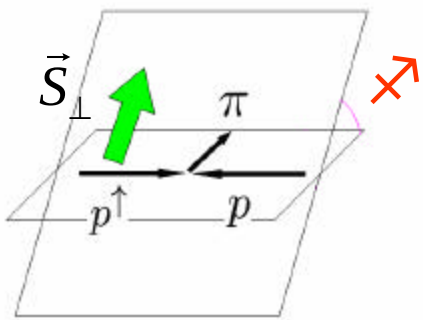
—— Other mechanisms necessary for a big SSA!

Origin of "T-Odd" observable

(De Rujula et al. NP,B35('71)365)

$A_N \sim \vec{S}_\perp \cdot \vec{p} \times \vec{p}_\pi \sim \sin\phi$ changes sign under $\vec{p} \to -\vec{p}$, $\vec{p}_\pi \to -\vec{p}_\pi$, $\vec{S}_\perp \to -\vec{S}_\perp$. $\longrightarrow$ called "T-odd".

"T-odd" $\neq$ "T-violating".



S-matrix for $i \to f$: $S_{fi} = \delta_{fi} + i(2\pi)^4\delta^4(P_f - P_i)T_{fi}$.

From Unitarity, $S^\dagger S = 1$: $T_{fi} - T^*_{if} = i\sum_n (2\pi)^4\delta^4(P_n - P_i)T_{ni}T^*_{nf} \equiv iA_{fi}$

A_{fi}: Absorptive part of T-matrix.

$$|T_{fi}|^2 = |T_{if}|^2 + 2\,\mathrm{Im}\{T_{fi}A^*_{fi}\} - |A_{fi}|^2.$$

Subtract $|T_{\tilde{f}\tilde{i}}|^2$ from both sides ($i = (\vec{P}, \vec{S}) \to \tilde{i} = (-\vec{P}, -\vec{S})$).

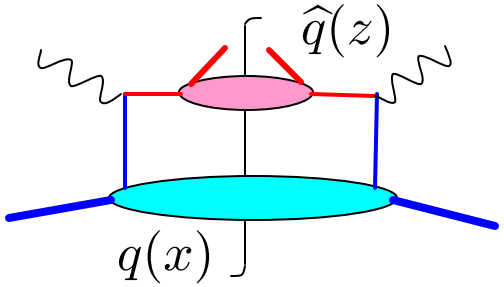
$$|T_{fi}|^2 - |T_{\tilde{f}\tilde{i}}|^2 = \underbrace{\cancel{\left(|T_{if}|^2 - |T_{\tilde{f}\tilde{i}}|^2\right)}}_{0} + 2\,\mathrm{Im}\{T_{fi}A^*_{fi}\} - |A_{fi}|^2.$$

By T-inv.

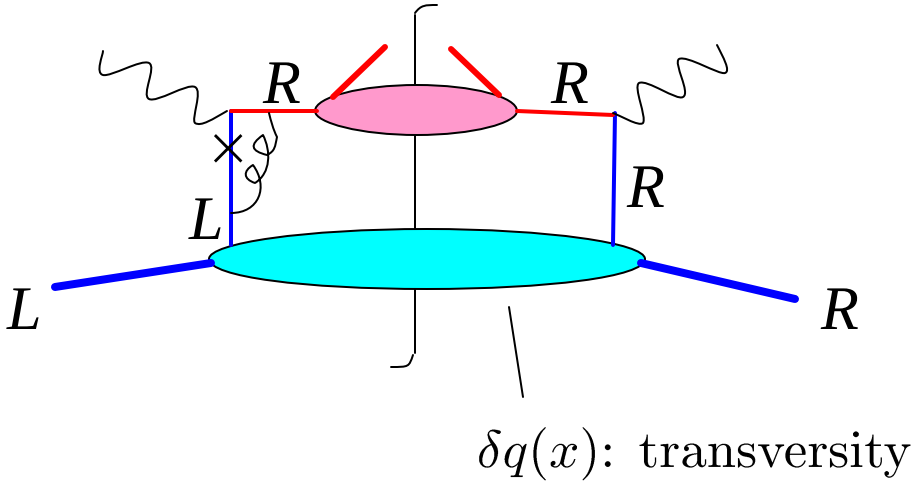
T-odd effect

QCD Factorization for $ep \to e\pi X$ (twist-2)

$$\sigma = q(x) \otimes \widehat{q}(z) \otimes \widehat{\sigma}_{\rm hard}$$



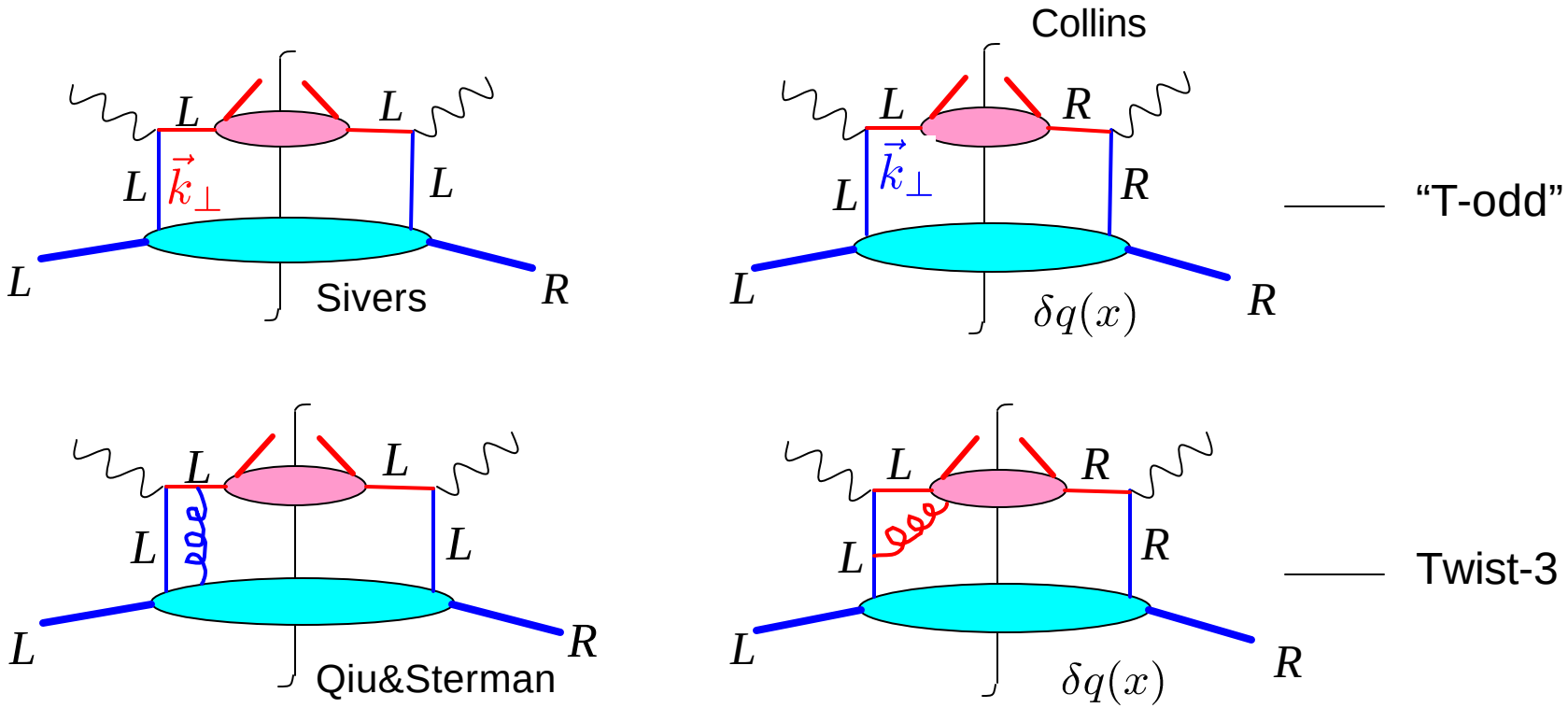
For $ep^{\uparrow} \to e\pi X$, we have



Usual twist-2:

suppressed by $\alpha_s m_q / p_T$

Implication by helicity flip: ex. SIDIS $ep^{\uparrow} \to e\pi X$:



- Helicity flip mechanism is proveded by introducing intrinsic $\vec{k}_\perp$ or explicit gluon field in the distribution/fragmentatiuon functions.
- The nonzero value of these functions is a consequence of the chiral-symmetry breaking.

2. "T-odd" distribution and fragmentation functions.

“T-odd” mechanism:

- Introduction of nonperturbative intrinsic k_T of partons.

  —— Intuitively understandable.

- Smooth start of SSA at low P_T:

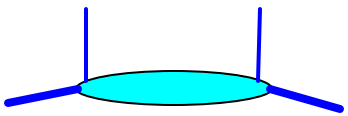
$$\mathrm{SSA} \sim \frac{M_N p_T}{p_T^2 + M_N^2}. \qquad \text{(Collins'93)}$$

- Factorization theorem ?

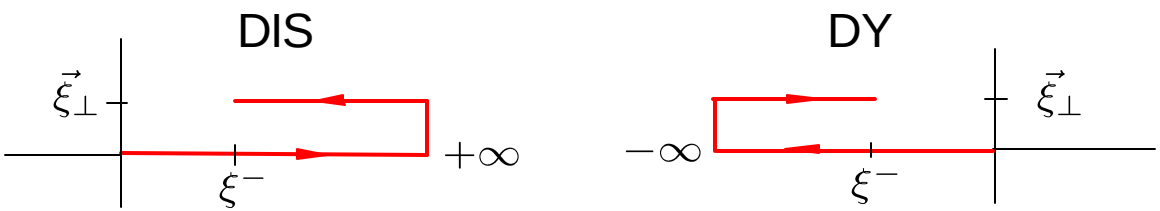
Distribution function with intrinsic $\vec{k}_\perp$. Ex. Sivers function (Sivers'90)

$$\int \frac{d\xi^- d^2\vec{\xi}_\perp}{(2\pi)^3} e^{ixP^+\xi^- - i\vec{k}_\perp\cdot\vec{\xi}_\perp} \langle PS|\bar{\psi}(0)U(0,\xi)\gamma^+\psi(\xi)|PS\rangle$$

$$= \quad 2q(x,\vec{k}_\perp) + \frac{2}{M_N} f_{1T}^\perp(x,\vec{k}_\perp)\epsilon_{\mu\nu\lambda\sigma}n^\mu p^\nu k_\perp^\lambda S_\perp^\sigma$$



$U(0,\xi)$: gauge-link op.



Remarks:

- $f_{1T}^\perp$: spin asymmetry of unpolarized quark dist. in $\perp$ pol. nucleon.
- $f_{1T}^\perp|_{\rm DY} = -f_{1T}^\perp|_{\rm DIS}$ by T-inv. of QCD. (Collins,('02); Belitsky et al.('03))
  — If gaugelink was ignored, $f_{1T}^\perp \equiv 0$. $\rightarrow$ called "T-odd".
- γ^+ projection: not suppressed by large scale. ("leading twist")
- M_N: scale of the chiral symmetry breaking. (~ 1 GeV)
- $\vec{k}_\perp$integration gives $\vec{\xi}_\perp = 0$. $\rightarrow \int d^2\vec{k}_\perp\, \vec{k}_\perp f_{1T}^\perp(x,\vec{k}_\perp) = 0$.
- Phase is provided by the exchange of gluons supplied by the gauge-link.

- "T-odd" fragmentation function. Ex. Collins function. (Collins '93)

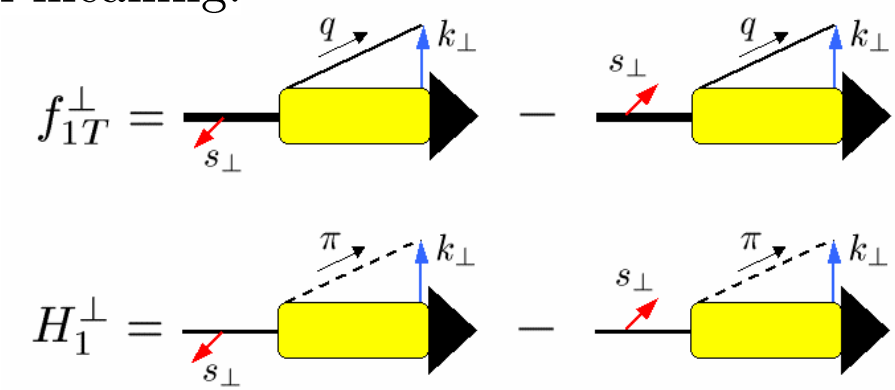
  For the case of fragmentation function, "T-odd" function is allowed by the final state interaction even without gauge-link.

$$\mathcal{F.T.}\sum_X \langle 0|U\gamma^+\gamma^\perp\gamma_5\psi(0)|\pi X\rangle_{\text{out}\,\text{out}}\langle \pi X|\bar{\psi}(\xi)U|0\rangle \sim \frac{1}{M_N}H_1^\perp(z,\vec{k}_\perp)\epsilon^{pnk_\perp S_\perp} + \cdots$$

  M_N: Scale for the chiral symmetry breaking. (Not m_π!)

  Universality not settled.

- Physical meaning:



- Other "T-odd" functions.(Boer, Mulders...)

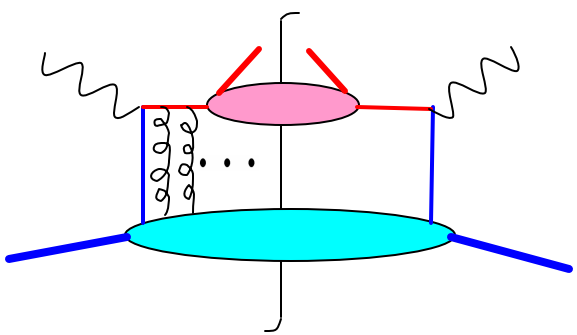
  $h_1^\perp(x,\vec{k}_\perp)$: distribution analogue of $H_1^\perp$.

  $D_{1T}^\perp(z,\vec{k}_\perp)$: fragmentation analogue of $f_{1T}^\perp$.

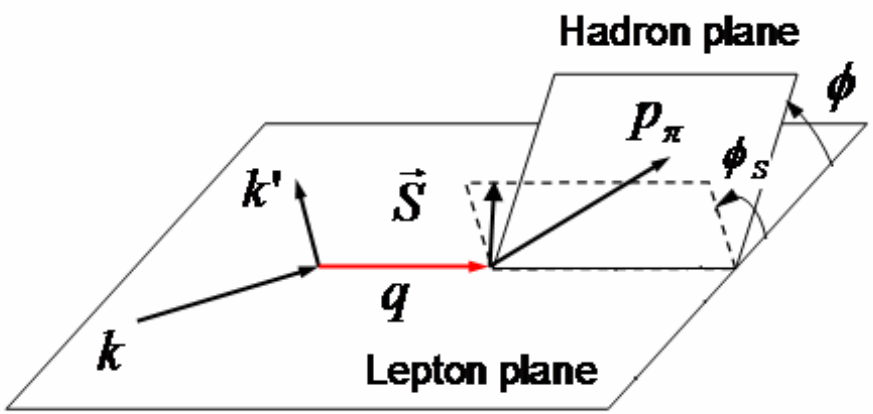
Applications of “T-odd” functions

- SIDIS $ep^\uparrow \to e\pi X$, Drell-Yan $p^\uparrow p \to \gamma^* X$ (Boer et al. NP B667 ('03) 201)

LO($O(\alpha_s^0)$) cross section was derived taking into account of gauge-link. It recovers the cross section obtained naively without including the gauge-link.



open problem: NLO?



Separation of Sivers and Collins effects:

$$\langle \sin(\phi - \phi_S) \rangle \sim \sum_a e_a^2 \, f_{1T}^{\perp a}(x, \vec{k}_\perp) \otimes \hat{q}^a(z) \otimes \hat{\sigma}_1,$$

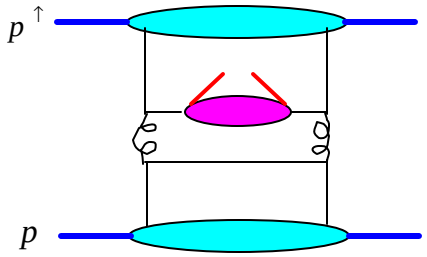
$$\langle \sin(\phi + \phi_S) \rangle \sim \sum_a e_a^2 \, \delta q^a(x) \otimes H_1^{\perp a}(z) \otimes \hat{\sigma}_2.$$

Applications of “T-odd” functions (continued)

- $p^{\uparrow}p \to \pi X$:

Phenomenology with Sivers effect (Sivers(’90),Anselmino et al. (’95)) and Collins effect (Anselmino et al (’99))

$O(\alpha_s)$ cross section without taking into account gauge-link.



Smooth start of A_N at $p_T = 0$, but eventually $1/p_T$ at large p_T. (Sivers’90)

Open problem: What happens with gauge link?

Phenomenology of $p^\uparrow p \to \pi X$ and $\bar{p}^\uparrow p \to \pi X$

Anselmino et al.('95,'98,'99), Boglione & Leader('00)

$$\begin{aligned}
\Delta\sigma \quad &\sim \quad f_{1T}(x,\vec{k}_\perp) \otimes q(x') \otimes \hat{q}(z) \otimes \hat{\sigma}_a \quad (a) \qquad \text{(Sivers)} \\
&+ \quad \delta q(x) \otimes q(x') \otimes H_1^\perp(z,\vec{k}_\perp) \otimes \hat{\sigma}_b \quad (b) \qquad \text{(Collins)} \\
&+ \quad \delta q(x) \otimes h_1^\perp(x',\vec{k}_\perp) \otimes \hat{q}(z) \otimes \hat{\sigma}_c \quad (c) \quad \text{——} \quad \text{Negligible.}
\end{aligned}$$

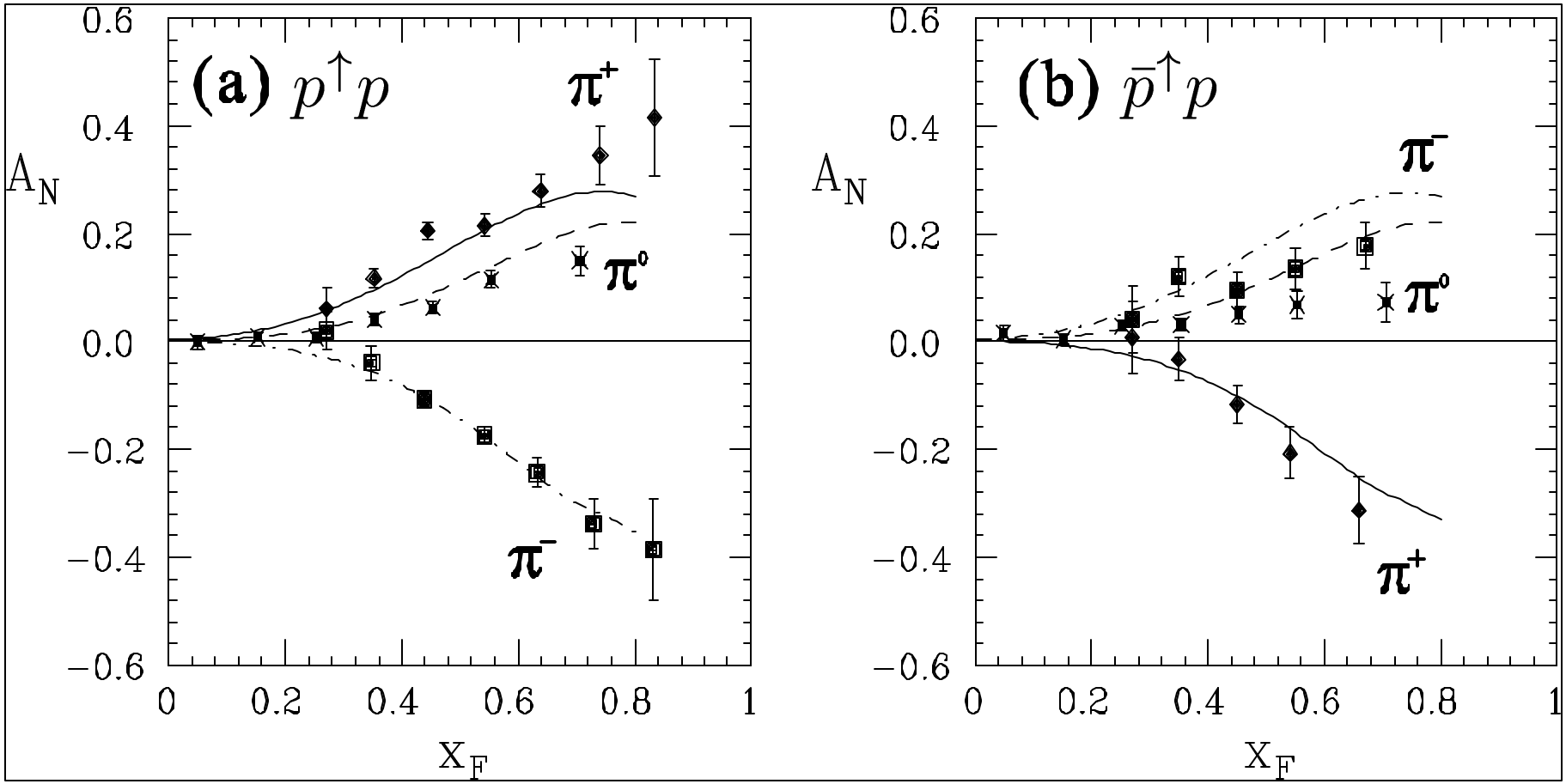
A_N at large x_F is governed by large (valence) region of x and z, and sea region of x'.

(a): From $A_N^{\pi^+} \simeq -A_N^{\pi^-}$, $f_{1T}^{\perp u}(x,\vec{k}_\perp) \simeq -f_{1T}^{\perp d}(x,\vec{k}_\perp)$. This leads to slightly smaller $A_N^{\pi^0}$ than $A_N^{\pi^+}$. For $\bar{p}^\uparrow p \to \pi X$, π^+ and π^- is interchanged.

(b): $H_1^{\perp u,\bar{d}}(\pi^+) = H_1^{\perp d,\bar{u}}(\pi^-)$ by charge conjugation and isospin sym. To reproduce $A_N^{\pi^\pm}$, $\delta u(x)/u(x) \simeq -0.76\,\delta d(x)/d(x)$. (Recall $\Delta u(x)/u(x) > |\Delta d(x)/d(x)|$.) This leads to slightly smaller $A_N^{\pi^0}$ than $A_N^{\pi^+}$. For $\bar{p}^\uparrow p \to \pi X$, π^+ and π^- is interchanged.

FNAL-E704: A_N for $p^\uparrow p \to \pi X$ and $\bar{p}^\uparrow p \to \pi X$

$\sqrt{s} = 20$ GeV, $p_T = 1.5$ GeV



$x_F = 2p_{\|}/\sqrt{s}$

To explain rising A_N at large x_F, rising $f_{1T}^{\perp}/q(x)$ and/or $H_1^{\perp}/D(z)$ necessary.

(b) violates Soffer's inequality seriously. (Boglione et al. (PRD61('00)114001.)

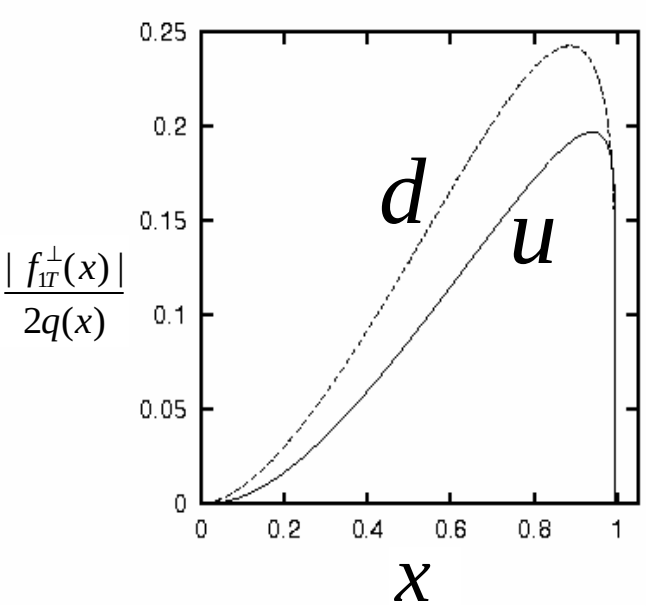
$$2|\delta q(x)| \leq q(x) + \Delta q(x)$$

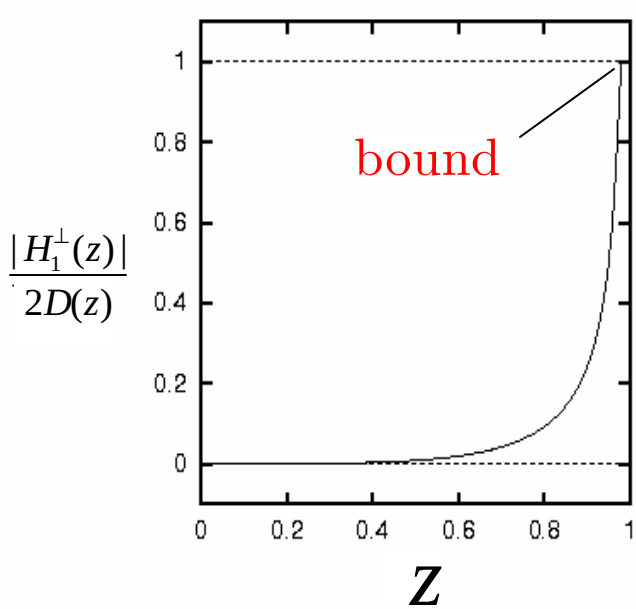
Only A_N at $x_F < 0.7$ can be reproduced.

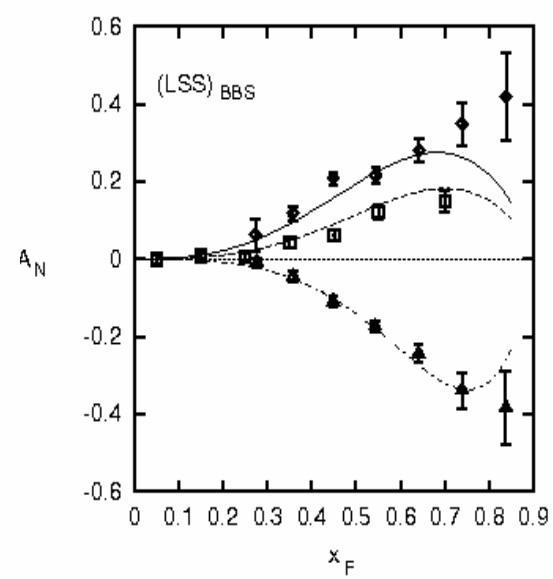
——— Saturation of A_N by Collins effect seems unlikely.

Anselmino et al, PRD60('99)054027

Boglione & Leader, PRD61('00)114001.







3. Twist-3 effects.

Twist-3 mechanism for SSA

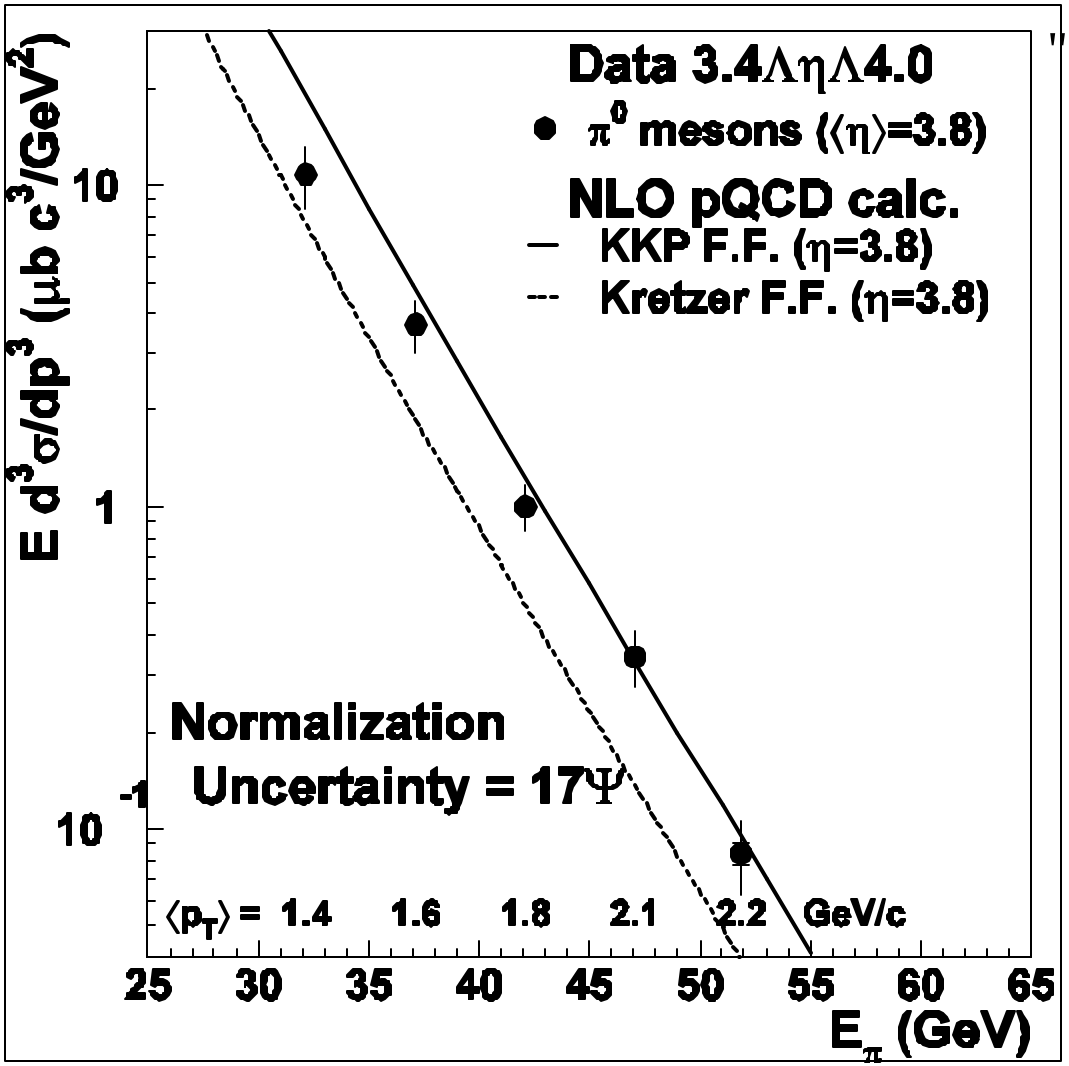
- Systematic approach based on collinear factorization
- P_T is produced solely from the partonic hard cross section

  ——— Suited for large-P_T production

  Note: In unpolarized $pp \to \pi X$, collinear factorization (twist-2) works as low as $P_T = 1.5$ GeV in the forward direction at $\sqrt{s} = 200$ GeV. (Vogelsang et al('03))

- SSA is a probe of quark-gluon correlation.
- Twist-3 functions are all "T-even".
- SSA is 1/Q suppressed: $O(\frac{M_N}{P_T})$ or $O(\frac{M_N P_T}{(-U)})$ in $p^{\uparrow}p \to \pi X$.
- At what P_T, does the data fall on twist-3 mechanism?

From STAR collab. hep-ex/0310058



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Factorization for twist-3: three twist-3 cross sections

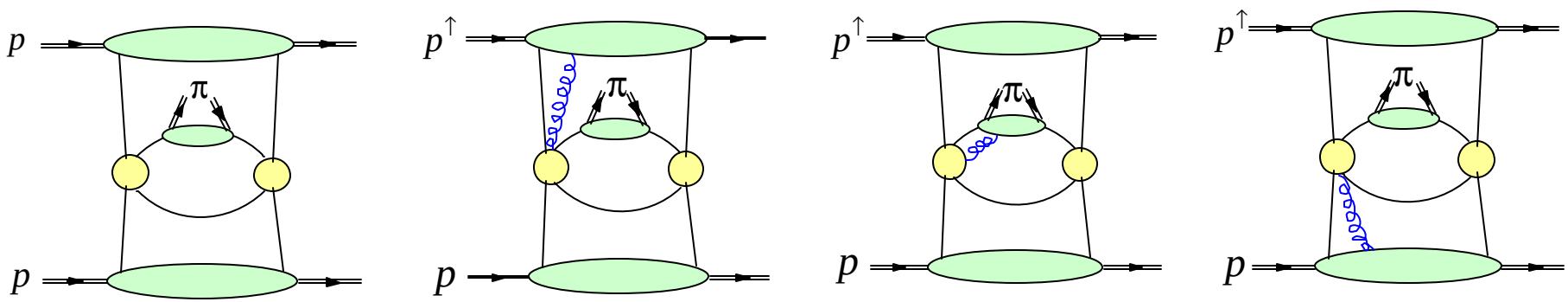
$$p^{\uparrow}(p, S_{\perp}) + p(p') \to \pi(\ell) + X,$$

$$\Delta\sigma \sim G_F(x_1, x_2) \otimes q(x') \otimes \hat{q}(z) \otimes \hat{\sigma}_A \quad (A)$$ (Qiu&Sterman '99)

$$+ \delta q(x) \otimes q(x') \otimes \widehat{E}_F(z_1, z_2) \otimes \hat{\sigma}_B \quad (B)$$ (Koike '02)

$$+ \delta q(x) \otimes E_F(x_1', x_2') \otimes \hat{q}(z) \otimes \hat{\sigma}_C \quad (C)$$ (Kanazawa&Koike '00)

δq: transversity



(A) (B) (C)

Unpolarized: twist-2

Polarized: twist-3

:Hard part

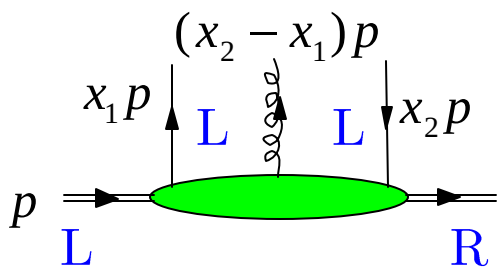
$$p^2 = n^2 = 0,\ p \cdot n = 1$$

Twist-3 distribtions:

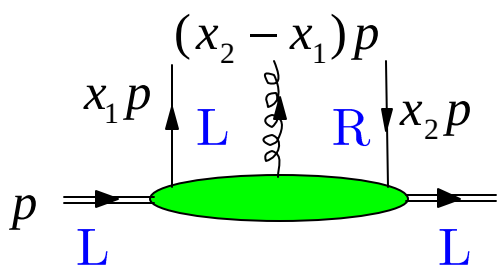
$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \langle PS | \bar{\psi}_j(0) g F^{\alpha\beta}(\mu n) n_\beta \psi_i(\lambda n) | PS \rangle$$
$$= \frac{M_N}{4} (\not{p})_{ij}\, \epsilon^{\alpha p n S_\perp} G_F(x_1, x_2) + \frac{M_N}{4} (\gamma_5 \not{p} \gamma_\nu)_{ij}\, \epsilon^{\nu \alpha n p} E_F(x_1, x_2) + \cdots$$

M_N: chiral symmetry breaking scale.

$G_F(x_1, x_2) \sim$



$E_F(x_1, x_2) \sim$



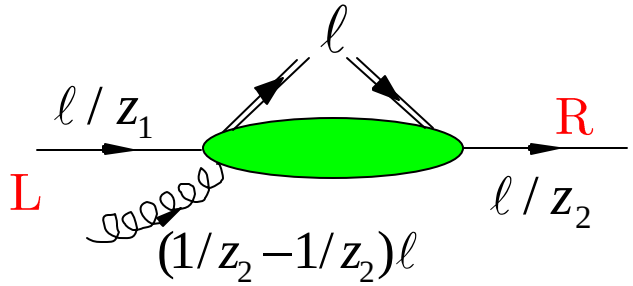
Twist-3 fragmentation function

$$\ell^2 = w^2 = 0,\ \ell \cdot w = 1$$

$$\frac{1}{N_c}\int \frac{d\lambda}{2\pi}\int \frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z_2}-\frac{1}{z_1})} \langle 0|\psi_i(0)|\pi(\ell)X\rangle\langle\pi(\ell)X|gF^{\alpha\beta}(\mu n)w_\beta\bar{\psi}_j(\lambda n)|0\rangle$$

$$= \frac{M_N}{2z_2}\left(\gamma_5 \not{\ell}\gamma_\nu\right)_{ij}\epsilon^{\nu\alpha w\ell}\widehat{E}_F(z_1,z_2) + \cdots$$

$\widehat{E}_F(z_1,z_2) \sim$



M_N: Scale for the chiral symmetry breaking.

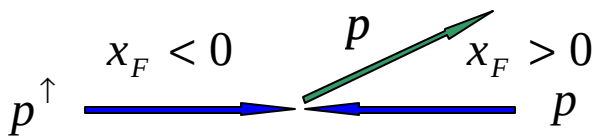
Characteristics of the twist-3 analysis

(cf. Qiu & Sterman '91 for p+p? ? ? +X)

- $G_F(x_1, x_2)$ and $\widehat{E}_F(z_1, z_2)$ contribute as <u>soft-gluon pole</u> (SGP) (i.e. $x_1 = x_2$, or $z_1 = z_2$), and, inparticular, their derivatives $\frac{d}{dx}G_F(x,x)$ and $\frac{d}{dz}\widehat{E}_F(z,z)$ also appear.

- Phase is provided as an imaginary part of the internal propagator $\frac{1}{x_1 - x_2 + i\epsilon} = P\frac{1}{x_1 - x_2} - i\pi\delta(x_1 - x_2)$, which leads to SGP.

- At $x_F \to 1$, main contribution is from the region with $x \to 1$, $x' \to 0$ and $z \to 1$.



$$\left| x\frac{d}{dx}G_F(x,x) \right| >> |G_F(x,x)|, \text{ as } G_F(x,x) \sim (1-x)^{\beta} \ (\beta > 0)$$
$$\left| z\frac{d}{dz}\widehat{E}_F(x,x) \right| >> |\widehat{E}_F(x,x)|$$

- Keep only terms with the derivatives $\frac{d}{dx}G_F(x,x)$ and $\frac{d}{dz}\widehat{E}_F(z,z)$ for valence quarks (Valence-quark soft-gluon approximation(Qiu&Sterman'99).

Characteristics of the twist-3 analysis (continued)

(A),(B): large contribution in the forward region ($x_F \to 1$) owing to (i) the derivatives $\frac{d}{dx}G_F(x,x)$ and $\frac{d}{dz}\widehat{E}_F(z,z)$, and (ii) the properties of $\hat{\sigma}_A$ and $\hat{\sigma}_B$.

$A_N \sim KM_N \left(O(\frac{p_T}{-U}) + O(\frac{1}{p_T})\right) \frac{1}{1-x_F} \sin\phi$: typical to twist-3.

Unphysical $\frac{1}{1-x_F}$ behavior is due the ansatz:
$G_F(x,x) \equiv Kq(x)$ and $\widehat{E}_F(z,z) \equiv \widehat{K}\widehat{q}(z)$ with a constant K and K'.

(A): $K_u = -K_d = 0.07$ to reproduce $A_N^{\pi^+} \simeq -A_N^{\pi^-}$.

(B): $\widehat{K}^{\pi^+}_{u,\bar{d}} = \widehat{K}^{\pi^-}_{d,\bar{u}} = -0.19$. $\delta u(x) = 0.58\Delta u(x)$ and $\delta d(x) = \Delta d(x)$.

(C): negligible due to the smallness of $\hat{\sigma}_C$.

x_F-dependence of A_N at E704 and RHIC energy

E704

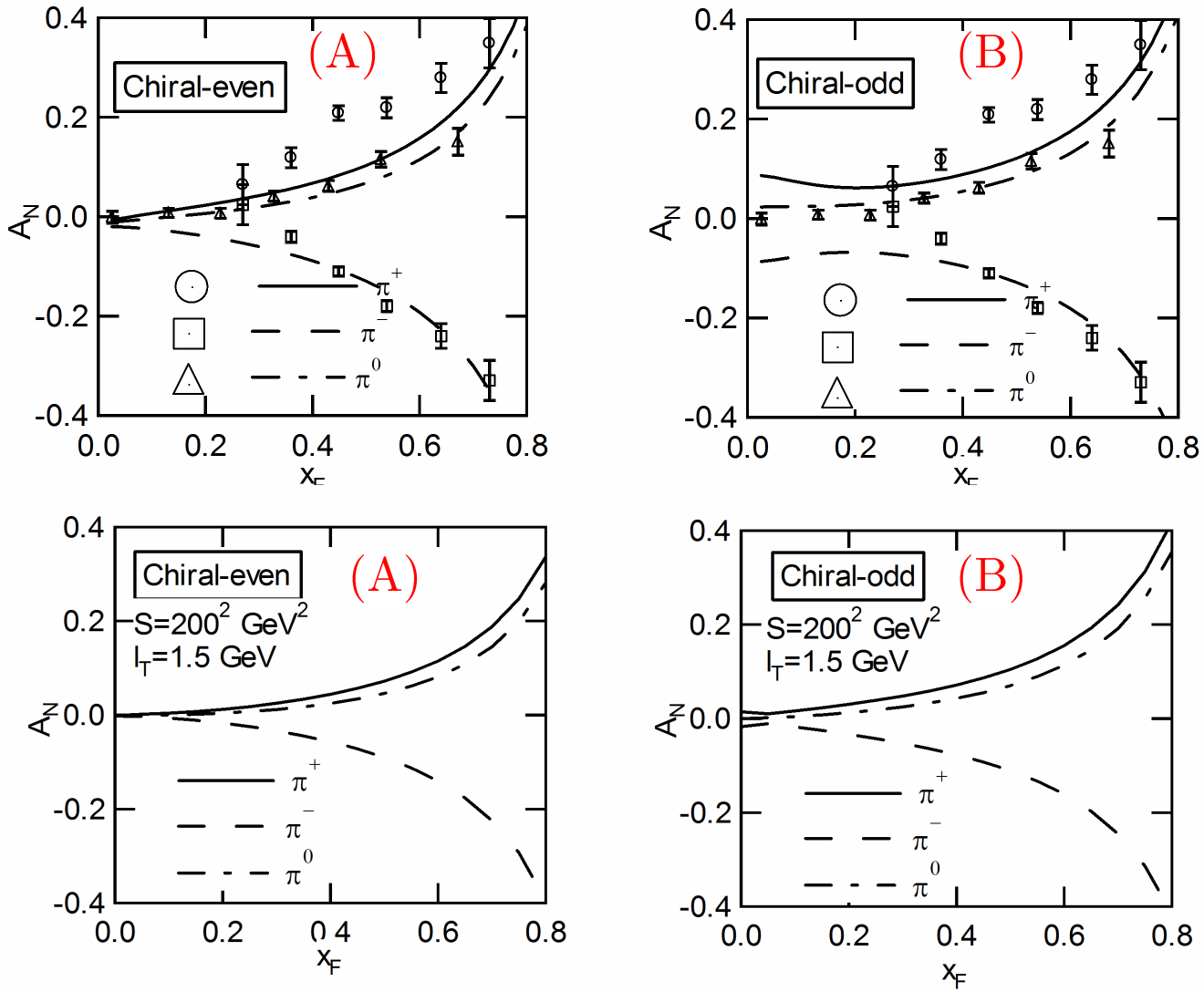
$\sqrt{S} = 20$ GeV

$\ell_T = 1.5$ GeV

RHIC STAR

$\sqrt{S} = 200$ GeV

$\ell_T = 1.5$ GeV

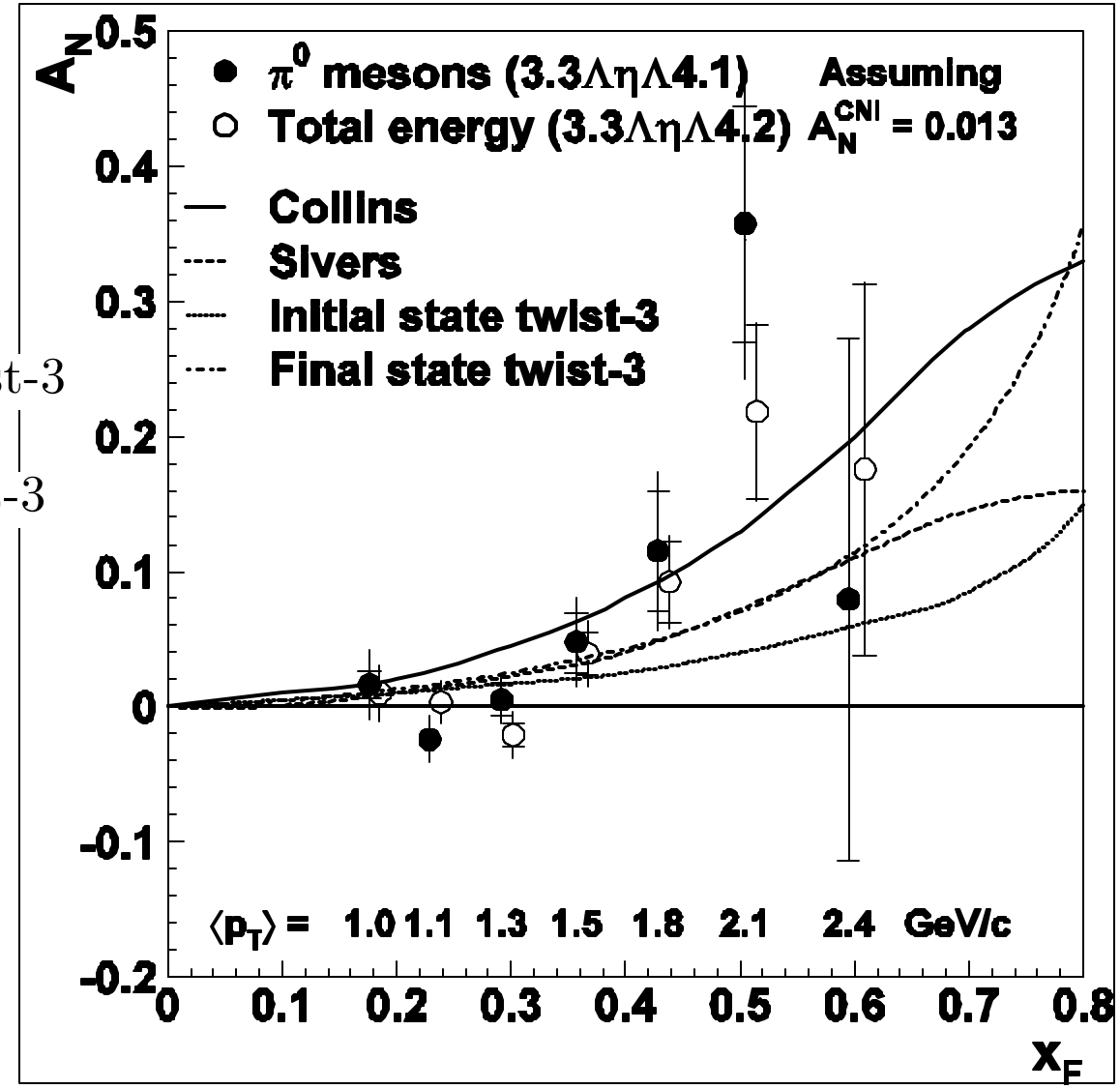


Chiral-even one; the same as Qiu & Sterman('99)

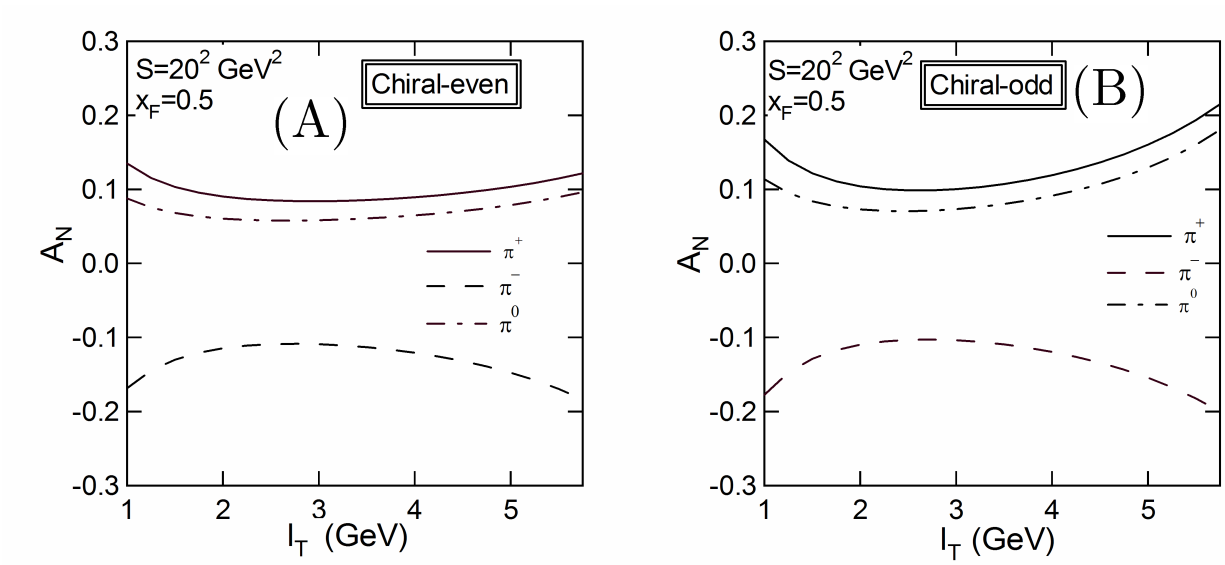
RHIC-STAR, hep-ex/0310068

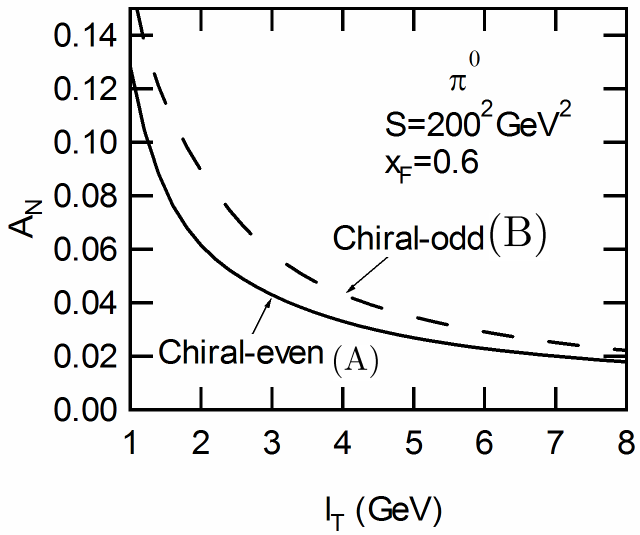
(A): Initial state twist-3

(B): Final state twist-3

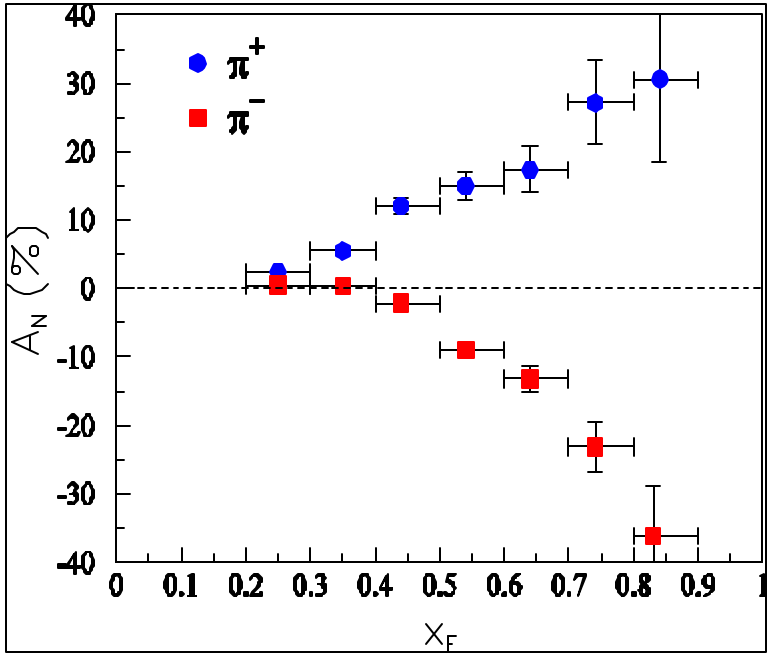


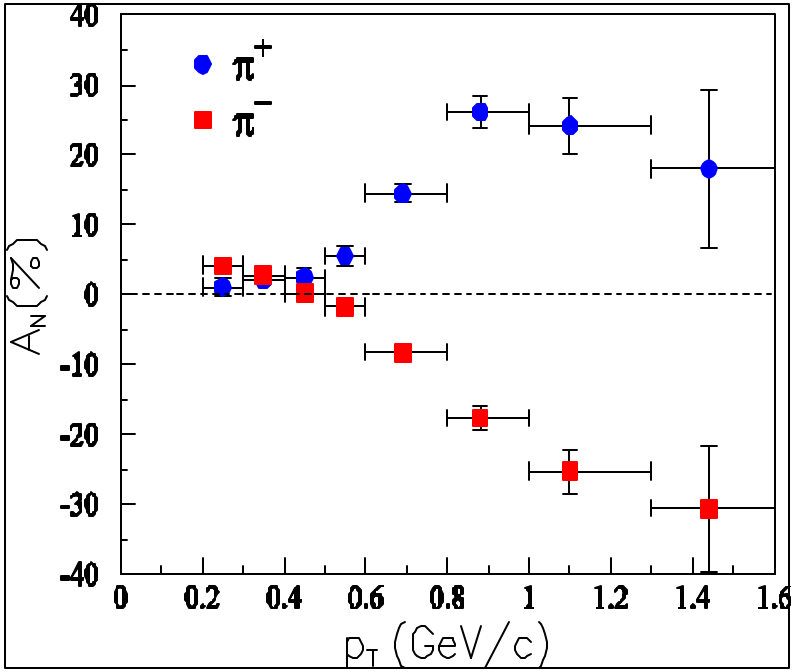
p_T-dependence of A_N





FNAL-E704: A_N for $p^{\uparrow}p \to \pi X$





P.L. B264 ('91) 462
P.L. B261 ('91) 201

A. Bravar, Proc. of Spin'96

- Connection between "T-odd" function and the SGP function. (Boer et al. NPB667('03)201)

$$\int d^2\vec{k}_\perp \, \vec{k}_\perp^2 f_{1T}^{\perp(+)}(x, \vec{k}_\perp) = M\pi G_F(x, x).$$

SGP describes the effect of parton's $\vec{k}_\perp$ in the form of soft-gluon-field.

- This equation can be used to test consistency between Sivers and SGP mechanism using E704+STAR and HERMES data.

- BUT, these are the relations among distrbution/fragmentation functions themselves. For the cross section (and SSA), more study is necessary.

Summary table of “T-odd” and Soft-gluon-pole (SGP) functions

T-odd function	SGP function	Physical meaning
$f_{1T}^{\perp}(x, \vec{k}_\perp)$	$G_F(x, x)$	$p^\uparrow \to q$
$\underline{h_1^{\perp}(x, \vec{k}_\perp)}$	$\underline{E_F(x, x)}$	$p \to q^\uparrow$
$D_{1T}^{\perp}(z, \vec{k}_\perp)$	$\widehat{G}_F(z, z)$	$q \to \Lambda^\uparrow$ etc
$\underline{H_1^{\perp}(z, \vec{k}_\perp)}$	$\underline{\widehat{E}_F(z, z)}$	$q^\uparrow \to \pi$ etc

——— : Chiral-odd

Summary and outlook

- A Big SSA is a consequence of the chiral symmetry breaking in hadron structure. Its effect can be systematically incorporated by "T-odd" functions and/or twist-3 functions.

- For SIDIS and Drell-Yan, LO SSAs have been derived in terms of Sivers & Collins functions with gaugelink operators. The result is the same as the naive one without gauge link. $\rightarrow$ What happens beyond LO? How about $p^{\uparrow}p \rightarrow \pi X$?

- Direct connection between Sivers function and SGP: $\int d\vec{k}_{\perp}^{2}\, \vec{k}_{\perp}^{2}\, f_{1T}^{\perp}(x, \vec{k}_{\perp}) = \pi M_N G_F(x, x)$.

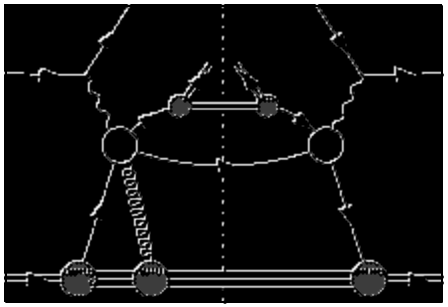
- $f_{1T}^{\perp}|_{\rm DIS} = -f_{1T}^{\perp}|_{\rm DY}$; BUT universality of "T-odd" fragmentation function?

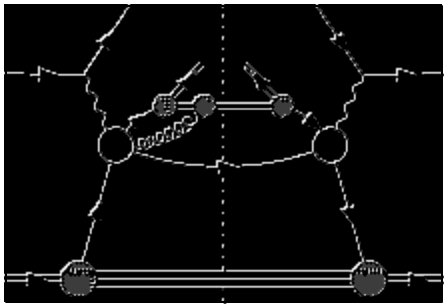
- At large p_T, the "T-odd" mechanism eventually leads to $1/p_T$-suppressed SSA (Sivers'90).

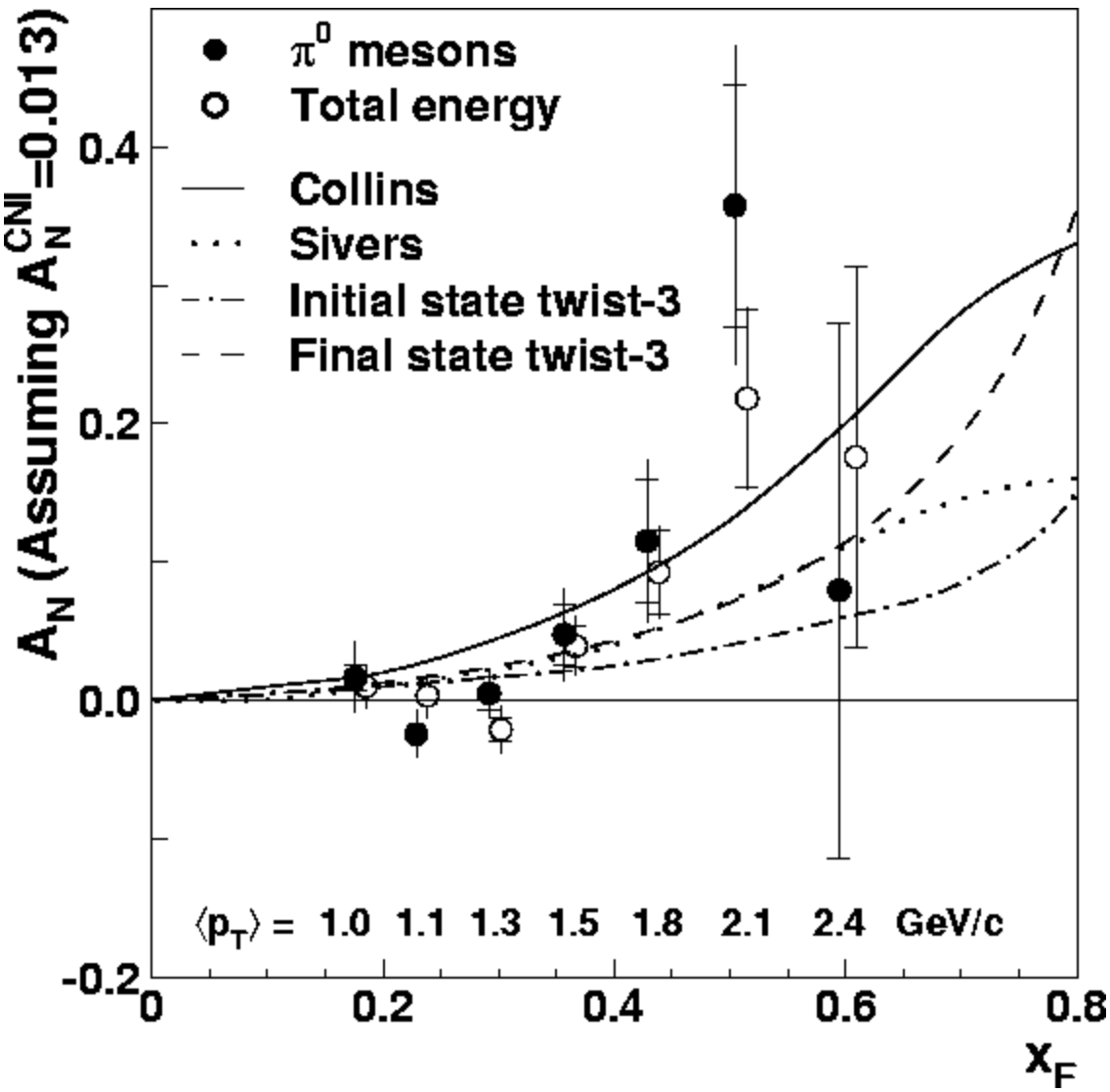
- Derivative of two SGP functions naturaly leads to observed rising A_N at large x_F. (No violation of Soffers' equality.)

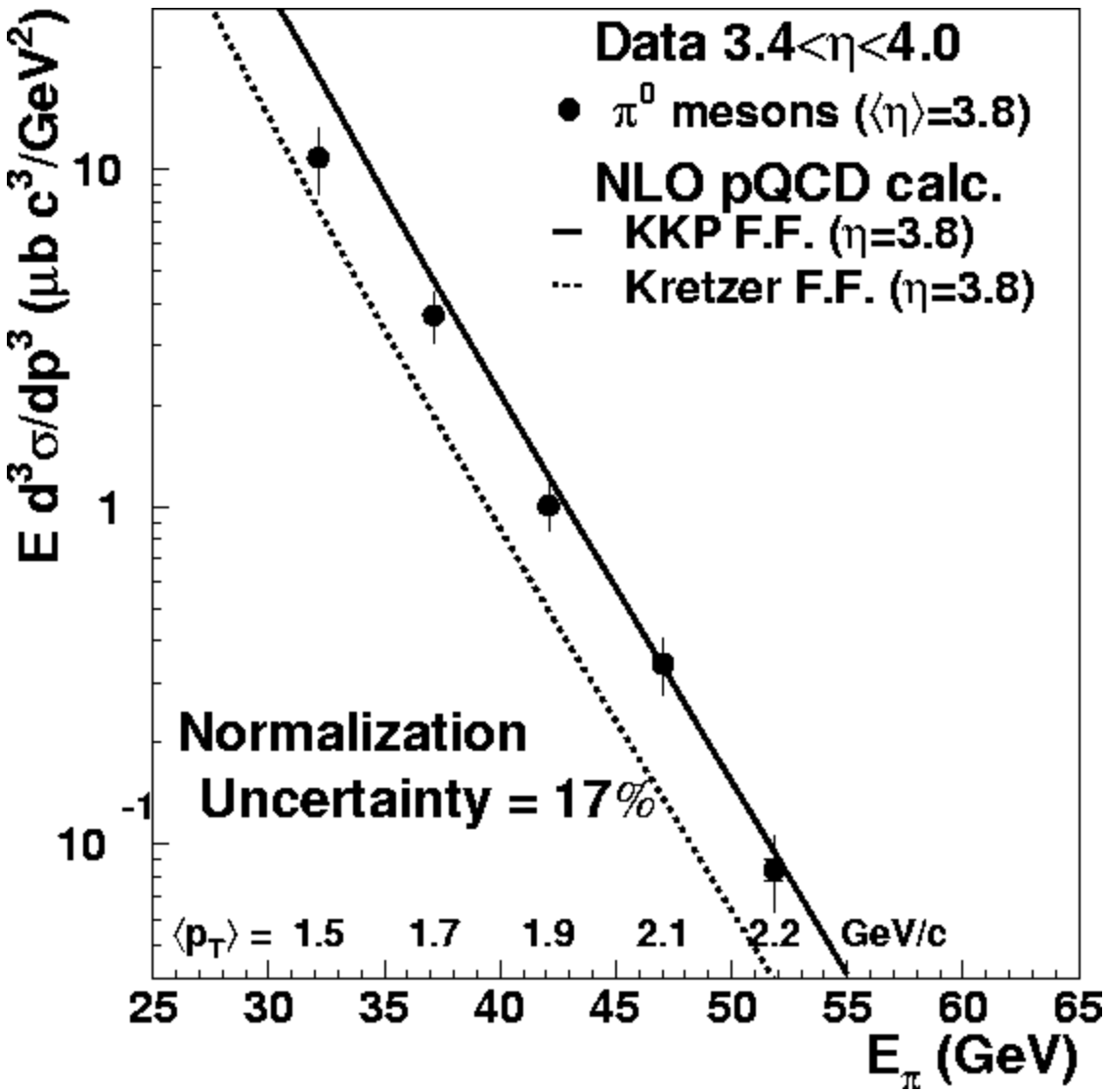
- At low p_T, twist-3 SSA blows up as $1/p_T$. $\rightarrow$ More data on p_T-dependence to see where the data starts to follow twist-3.

- Twist-3 mechanism can be extended to large p_T hadron production in SIDIS relevant to eRHIC.






π^0 mesons
Total energy
Collins
Sivers
Initial state twist-3
Final state twist-3
A_N (Assuming A_N^{CNI}=0.013)
0.4
0.2
0.0
-0.2
0
0.2
0.4
0.6
0.8
x_F
$\langle p_T \rangle$ = 1.0 1.1 1.3 1.5 1.8 2.1 2.4 GeV/c


Data 3.4<η<4.0
π0 mesons (⟨η⟩=3.8)
NLO pQCD calc.
KKP F.F. (η=3.8)
Kretzer F.F. (η=3.8)
Normalization
Uncertainty = 17%
⟨pT⟩ = 1.5 1.7 1.9 2.1 2.2 GeV/c
E d3σ/dp3 (μb c3/GeV2)
Eπ (GeV)
10
1
10-1
25 30 35 40 45 50 55 60 65